

# Multilevel Parallelization Models: Application to VIV

S. Dong, D. Lucor, V. Symeonidis, J. Xu, and G.E. Karniadakis  
Brown University, Providence, RI  
gk@dam.brown.edu

## Abstract

*Realistic simulations of flow past a flexible cylinder subject to vortex-induced vibrations require a large number of Fourier modes along the cylinder span and high resolutions in the streamwise and cross-flow directions. Parallel computations employing a single-level parallelism for this type of problems have clear performance limitations that prevent effective scaling to the large processor count on modern supercomputers. In this paper we present two multilevel parallel paradigms based on MPI/MPI and MPI/OpenMP for high-order CFD methods within the spectral element framework and compare their performance. In the MPI/MPI model, we employ MPI process groups/communicators to decompose the flow domain and MPI processes into different levels. In the MPI/OpenMP model, we employ multiple OpenMP threads to split the workload within the sub-domain and take a coarse-grain approach that significantly reduces the OpenMP synchronizations. For identical configurations the MPI/MPI model is observed to be generally more efficient. However, for dynamic p-refinement the MPI/OpenMP approach is more effective. Because a greatly reduced number of processes are involved in the communications at each level, these multilevel parallel paradigms reduce the network latency overhead and enable the applications to scale to a large number of processors more efficiently.*

## 1. Vortex Induced Vibration

Flow past a flexible cylinder subject to vortex-induced vibrations (VIV) arises in many industrial and military situations, such as the flexible risers and tendons in petroleum production, power lines, heat exchangers, and marine tow cables<sup>(see [2, 10, 20])</sup>. Flows in these applications are often sheared, which can potentially excite a large number of modes. How many and what modes are excited in a particular flow situation can significantly affect the durability and lifespan of the structure<sup>[15]</sup>.

However, predication of vortex-induced vibrations has been based on semi-empirical methods until recently. Most of the models involve eigen-solutions of the structures, which depend on the values of drag and lift coefficients obtained empirically, either the sectional values of the span-averaged values. When these models predict several modes as possible candidates to respond, it is not certain whether the structures would respond in all of these modes—it may transition from one mode to another or from one set of modes to another set of modes.

Recently, direct numerical simulation (DNS) has been applied to study the flow past a flexible cable subject to VIV<sup>[6, 7, 17, 18]</sup>, and offers accurate predictions of both flow and structural quantities. In DNS of a laminar flow past a three-dimensional freely vibrating cable, Newman and Karniadakis<sup>[18]</sup> employed a simple wave equation to model the motion of the structure, neglecting the effect of bending stiffness. Two possible states of the wake were observed in their simulations: one that corresponds to a traveling wave response, and another one that corresponds to a standing wave response. Direct numerical simulations of a turbulent flow past a flexible cylinder<sup>[6]</sup> showed that the structural oscillation demonstrates much higher amplitude in the turbulent regime. When the bending stiffness is taken into account, significant differences are observed in both the flow structures and dynamics of the cylinder.

## 2. Inherent Hierarchical Structures in VIV Computations

We consider the incompressible flow past a long flexible cylinder subject to vortex-induced vibrations. The equations that describe this problem are the coupled system of fluid equations and cylinder's structure equations. The Navier-Stokes equations and the continuity equation are the governing equations for the fluid motion,

$$\frac{\partial \vec{u}'}{\partial t} + (\vec{u}' \cdot \nabla) \vec{u}' = -\frac{1}{\rho_f} \nabla' p' + \frac{1}{\text{Re}} \nabla'^2 \vec{u}', \quad (1)$$

$$\nabla' \cdot \vec{u}' = 0, \quad (2)$$

where  $\vec{u}'$  is the velocity field,  $p'$  is the pressure,  $\text{Re} = UD/\nu$  is the Reynolds number based on the free-stream velocity  $U$ , cylinder diameter  $D$  and kinematic viscosity  $\nu$ ,  $\rho_f$  is the fluid density, and  $\nabla'$  is the gradient operator in the inertial coordinate system. We only consider the cross-flow motion of the structure which is described by the following equation:

$$\rho \frac{\partial^2 Y}{\partial t^2} - c^2 \frac{\partial^2 Y}{\partial z^2} + \gamma^2 \frac{\partial^4 Y}{\partial z^4} + \frac{4\pi\zeta}{U_r} \frac{\partial Y}{\partial t} + \left( \frac{2\pi}{U_r} \right) Y = \frac{1}{2} \frac{F_L}{m}, \quad (3)$$

where we denote by  $Y(z,t)$  the non-dimensional cross-flow displacement that has been normalized by the cylinder diameter  $D$ . The damping fraction is  $\zeta$  and  $U_r = U/fD$  is the reduced velocity. The mass ratio and the natural frequency of the structure are  $m$  and  $f$ , respectively. The free-stream velocity of the inflow  $U$  is taken as the reference velocity of the system. Also,  $c = \sqrt{T/\rho_s} U^2$

and  $\gamma = \sqrt{EI/\rho_s} U^2 D^2$  (where  $\rho_s$  is the structural density) are the non-dimensional cable/beam phase velocities respectively, where  $T$  is the tension and  $EI$  is the bending stiffness. The total lift force  $F_L(z,t)$  is obtained through the flow solver, and is computed by integrating the pressure and viscous terms along the span of the cylinder

$$F_L = \vec{j} \cdot \oint (-p' \vec{n} + \nu (\nabla \vec{u}' + \nabla \vec{u}'^T) \cdot \vec{n}) ds, \quad (4)$$

where the integration is performed around the circumference of the cylinder at each spanwise location,  $\vec{n}$  is the outward unit vector normal to the cable, and  $\vec{j}$  is the unit vector in the cross-flow direction.

Solving the fluid/structure interaction problems generally involves moving the computational domains and dynamic re-meshing. However, we can eliminate the difficulty of a moving mesh by using body-fitted coordinates with the coordinate axes attached to the flexible cable<sup>[3]</sup>. The transformation maps the time-dependent and deforming domain to a stationary and non-deforming one. This mapping is described by the following transformation:

$$y = y' - Y(z,t). \quad (5)$$

In the transformed system of coordinates the flexible cylinder appears as straight and stationary. The Navier-Stokes equation and the continuity equation are transformed as follows:

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla) \vec{u} = -\nabla' p + \frac{1}{\text{Re}} \nabla^2 \vec{u} + \vec{A}(\vec{u}, p, Y), \quad (6)$$

$$\nabla \cdot \vec{u} = 0, \quad (7)$$

where  $\vec{A}(\vec{u}, p, Y)$  is the additional acceleration introduced by the non-inertial transformation (5). The coupled fluid/cylinder problem is solved in three steps. The fluid equations are first solved for the given cylinder motion  $Y(z,t)$ . The lift force on the cable is then computed as a function of  $z$ . The cylinder motion is updated finally.

Such VIV computations demonstrate inherent hierarchical structures when the problem is discretized with spectral element high-order methods. For the flow/cylinder problem the flow velocity can be represented by

$$u(x, y, z, t) = \sum_k \hat{u}_k^*(x, y, t) e^{ikz} \quad (8)$$

This representation assumes that the flow and cylinder variables are periodic in the spanwise direction with a period equal to the cylinder length  $L$ . A combined spectral element-Fourier discretization<sup>[12]</sup> can be employed to accommodate the requirements of high-order as well as efficient handling of multiply connected computational domain in the non-homogeneous planes. Spectral expansions in the homogeneous direction employ Fourier modes that are decoupled except in the nonlinear terms. Each Fourier mode is 2D field of space and time, and can be solved with the spectral element approach.

Specifically, a straight-forward mapping of the Fourier modes onto the processors can be employed. This results in an efficient and balanced computation where the three-dimensional problem is decomposed into two-dimensional problems using multiple one-dimensional