Evaluation of Parallelized Meshless Approach: Application to Shielding Current Analysis in HTS

Soichiro Ikuno[1], Toshihiro Hanawa[2], Teruou Takayama[3], and Atsushi Kamitani[3]

[1]School of Computer Science, Tokyo University of Technology, Hachioji, Tokyo 192-0982, Japan
[2]Center for Computational Science, University of Tsukuba, 305-8577 Ibaraki, Japan
[3]Department of Informatics, Faculty of Engineering, Yamagata University, Yamagata 992-8510, Japan

Parallelized meshless approach using OpenMP is evaluated. The meshless approach does not require finite elements. However, it takes tremendous CPU time to generate the coefficient matrix of linear system instead of unnecessary mesh generation procedure. The code for Poisson problem and shielding current analysis in high-temperature superconductor (HTS) by using element-free Galerkin method is developed and parallelize the code using OpenMP. Results of the computation shows that the CPU time of 4PUs (processing units) is 3.8 times faster than that of 1PU in case of Poisson problem. However, speedup ratio does not show good performance in case of the shielding current analysis in HTS.

***Index Terms*—Element-free Galerkin, high-temperature superconductor (HTS), meshless approach, openMP, parallelization.**

I. INTRODUCTION

THE MESHLESS approach does not require finite elements of a geometrical structure. The necessary information is only locations of nodes which are scattered in the region and on the boundary. The meshless approaches have been developed, such as the element-free Galerkin (EFG) method [1] and the meshless local Petrov-Galerkin (MLPG) method. And these methods are applied to a variety of engineering fields and the fields of computational magnetics [2]. However, it takes tremendous CPU time to generate the coefficient matrix of linear system instead of unnecessary mesh generation procedure.

High-temperature (T_c) superconductors (HTSs) have various problems to solve in order to compete with presently used normal conductors. AC loss is one of such problems. However, even small amount of energy dissipation, which is caused by ac loss, will lead to critical raise of temperature in an HTS in superconductive state.

The time-dependent spatial distribution of the shielding current density in an HTS is necessary information to calculate the ac loss. Formulation of the electromagnetic behavior of the shielding current density in an HTS gives a system of time-dependent integro-differential equation. After discretizing the system by use of EFG and the completely implicit method, we obtain a nonlinear system. However, improper integrals appear as coefficients of the system and the integrand has a stronger singularity with decreasing thickness of the HTS.

The purpose of the present study is to develop numerical code for analyzing shielding current density in HTSs thin film using the EFG. The double exponential formula is applied to calculate improper integrals accurately. Furthermore, we parallelize EFG method using OpenMP to reduce the CPU time for calculating the problem.

II. SHIELDING CURRENT DENSITY ANALYSIS

Let us first explain the governing equation of shielding current density in YBCO superconductor thin film. The HTS thin film is placed in homogeneous ac magnetic field with its flux parallel to the thickness direction. For simplicity, we assume that the HTS thin film is disk-shaped whose radius is $R = 20$ mm and whose thickness is $D = 2\varepsilon$, and the area of the circular cross-section is constant along the thickness direction. Thus, we can treat the problem as axisymmetric. Further, we assume that the shielding current density does not flow along the c-axis because thickness is very thin compared with the radius of HTS.

Throughout this paper, let us use the cylindrical coordinate (r, θ, z) by taking the symmetry axis of the HTS as the z-direction. In terms of the coordinate, the applied magnetic flux density $\boldsymbol{B}_0$ is written as

$$\boldsymbol{B}_0 = B_0 \sin 2\pi f t \boldsymbol{e}_z$$

where B_0 denotes amplitude of magnetic flux density and f denotes frequency. $\boldsymbol{e}_z$ denotes the unit vector in the z-direction. Under these assumptions, there exists a scalar function $S(r,t)$ such that the shielding current density $\boldsymbol{j}$ satisfies

$$\boldsymbol{j} = \frac{1}{\varepsilon}\nabla S \times \boldsymbol{e}_z. \tag{1}$$

Using this scalar function, the behavior of the shielding current density can be expressed by the following time-dependent integro-differential equation:

$$\mu_0 \frac{\partial}{\partial t}\left(\int_0^R Q(r,r')S(r',t)r'dr' + \frac{S}{\varepsilon}\right) = -\frac{\partial(\boldsymbol{B}_0 \cdot \boldsymbol{e}_z)}{\partial t} - (\nabla \times \boldsymbol{E}) \cdot \boldsymbol{e}_z \tag{2}$$

where μ_0 denotes the magnetic permeability of vacuum. The function $Q(r,r')$ is defined as

$$Q(r,r') = \frac{-1}{4\pi\varepsilon^2\sqrt{rr'}} \sum_{m=0}^{1}\sum_{n=0}^{1}(-1)^{m+n}k^{mn}\mathrm{K}(k^{mn}) \tag{3}$$

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where $\mathrm{K}(x)$ denotes x's complete elliptic integral of the first kind, and k^{mn} is defined by

$$(k^{mn})^2 = \frac{4rr'}{(r+r')^2 + 4\varepsilon^2(1-\delta_{mn})} \tag{4}$$

where δ_{mn} denotes the Kronecker's delta. For the initial and boundary conditions to (2), we assume $S(r,0) = S(R,t) = 0$.

The initial-boundary-value problem of (2) is solved with the $J - E$ constitutive relation which is defined as following equation:

$$\boldsymbol{E} = E\left(|\boldsymbol{j}|\right)\frac{\boldsymbol{j}}{|\boldsymbol{j}|}. \tag{5}$$

Here, we adopt the flux-flow and flux-creep model in which the function $E(j)$ can be written in the form

$$E\left(|\boldsymbol{j}|\right) = \begin{cases} E_{\mathrm{c}}\frac{\sinh[\alpha(|\boldsymbol{j}|/j_{\mathrm{c}})]}{\sinh\alpha}; & |\boldsymbol{j}| \le j_{\mathrm{c}} \\ E_{\mathrm{c}} + \rho_{\mathrm{f}}\left(|\boldsymbol{j}| - j_{\mathrm{c}}\right); & |\boldsymbol{j}| > j_{\mathrm{c}} \end{cases} \tag{6}$$

where E_{c}, j_{c}, and ρ_{f} denote the critical electric field, the critical current density, and the flow resistivity, respectively. Furthermore, the constant α is defined by $\alpha \equiv U_0/(k_{\mathrm{B}}T)$, where U_0, k_{B}, and T denote the pinning potential, the Boltzmann constant, and the temperature, respectively.

The behavior of the shielding current density in HTSs thin film can be determined by solving the initial boundary value problem of (2). Throughout the present study, the physical and the geometric parameters are fixed as follows: $R = 20$ mm, $T = 77$ K, $j_{\mathrm{c}} = 1.0 \times 10^{10}$ A/m^2, and $E_{\mathrm{c}} = 0.1$ mV/m.

III. Discretization

Let us first scatter N nodes on the analytic domain and the boundary, and divide the analytic domain into the cell of N_{cell} piece. By applying the backward Euler method to (2), the system is discretized with respect to time, and it is transformed to the boundary-value problem. The problem is expressed as a weak form, and it is discretized with respect to space by using the EFG method [1]. Then we can obtain the following nonlinear system:

$$C\boldsymbol{s}^n + \frac{1}{\varepsilon}A\boldsymbol{s}^n - \Delta t\boldsymbol{e}^n + \lambda\boldsymbol{\phi} = C\boldsymbol{s}^{n-1} + \frac{1}{\varepsilon}A\boldsymbol{s}^{n-1} - \boldsymbol{b}$$

where the (i,j) element of matrices C and A are defined by

$$(C)_{ij} = \int_0^R r dr \int_0^R r'\phi_i\phi_j Q(r,r')dr'$$

$$(A)_{ij} = \int_0^R r\phi_i\phi_j dr$$

and vectors $\boldsymbol{e}$, $\boldsymbol{\phi}$, and $\boldsymbol{b}$ are defined as follows:

$$\begin{aligned} \boldsymbol{e} &= [e_1, e_2, \cdots, e_N]^T \\ \boldsymbol{\phi} &= [\phi_1, \phi_2, \cdots, \phi_N]^T \\ \boldsymbol{b} &= \left(B_0^n - B_0^{n-1}\right)[b_1, b_2, \cdots, b_N]^T \end{aligned}$$

Here, the superscript n denotes the time step and λ denotes the Lagrange multiplier. Furthermore, ϕ_i denotes the shape functions which is obtained by the moving least squares (MLS) approximation [1]. The elements of vectors $\boldsymbol{e}$ and $\boldsymbol{b}$ are defined as follows:

$$(e_i)^n = \int_0^R rE_\theta^n \frac{d}{dr}\phi(r)dr, b_i = \int_0^R r\phi dr.$$

Then we can obtain the following nonlinear system:

$$\begin{aligned} \boldsymbol{G}(\boldsymbol{s},\lambda) \equiv \begin{bmatrix} W & \boldsymbol{\phi} \\ \boldsymbol{\phi}^T & 0 \end{bmatrix}\begin{bmatrix} \boldsymbol{s}^n \\ \lambda \end{bmatrix} - \Delta t\begin{bmatrix} \boldsymbol{e}^n(\boldsymbol{s}) \\ 0 \end{bmatrix} - \begin{bmatrix} \boldsymbol{u} \\ 0 \end{bmatrix} \\ = \boldsymbol{0} \end{aligned} \tag{7}$$

where W and $\boldsymbol{u}$ are defined by $W = C + A/\varepsilon$, $\boldsymbol{u} = W\boldsymbol{s}^{n-1} - \boldsymbol{b}$, respectively. The scalar function S for each time step can be determined by solving the nonlinear system $\boldsymbol{G}(\boldsymbol{s},\lambda) = \boldsymbol{0}$ and substituting the scalar functions to (1), we can obtain the shielding current density.

In the discretizing process, we must numerically evaluate the following integrals on each cell to determine the matrix W:

$$\gamma_{ij} = \int_{r_1}^{r_2} dr \int_{r_1}^{r} dr'\Gamma_{ij}(r,r') \qquad (i,j = 1,2,\cdots,N) \tag{8}$$

where r_1 and r_2 denote the edge points of each cell. The function $\Gamma_{ij}(r,r')$ is given by

$$\Gamma_{ij}(r,r') = rr'Q(r,r')\left[\phi_i(r)\phi_j(r') + \phi_j(r)\phi_i(r')\right]. \tag{9}$$

The value of k^{mn} becomes a unity with the condtion $r = r'$, as is well known that the value of the complete elliptic integral $\mathrm{K}(x)$ diverges at $x = 1$. That is to say, the function $\Gamma_{ij}(r,r')$ becomes singular for $r = r'$. From the result, (8) becomes an improper integral. In addition, the integrand has a stronger singularity with decreasing thickness of the HTS.

When the Gauss-Legendre formula is applied to (8) for numerical evaluation, it is transformed to following equation:

$$\gamma_{ij} = \int_{-1}^{1} d\xi \int_{-1}^{1} d\eta G_{ij}(\xi,\eta) \tag{10}$$

where function $G_{ij}(\xi,\eta)$ is defined by

$$G_{ij}(\xi,\eta) = \frac{(r_2 - r_1)^2}{4}N_2(\xi)\Gamma_{ij}\left(r(\xi), r'\left(\xi'(\eta)\right)\right). \tag{11}$$

Here, $N_l(\xi)$, $\xi'(\eta)$, $r(\xi)$, and $r'(\xi)$ are given by

$$N_l(\xi) = \frac{1 + (-1)^l\xi}{2}, \xi'(\eta) = -N_1(\eta) + \xi N_2(\eta)$$

$$r(\xi) = \sum_{l=1}^{2} r_l N_l(\xi), r'(\xi) = \sum_{l=1}^{2} r_l N_l\left(\xi'(\eta)\right).$$

As is mentioned above, (8) becomes an improper integral for $r = r'$. This character is succeeded as it is, even if it transforms to (10). Therefore, the values of the function $G_{ij}(\xi,\eta)$ diverges for $\eta = 0$, and the accuracy of the (10) becomes very poor. When the Gauss-Legendre formula is actually adopted for the numerical evaluation for the matrix W, the following results

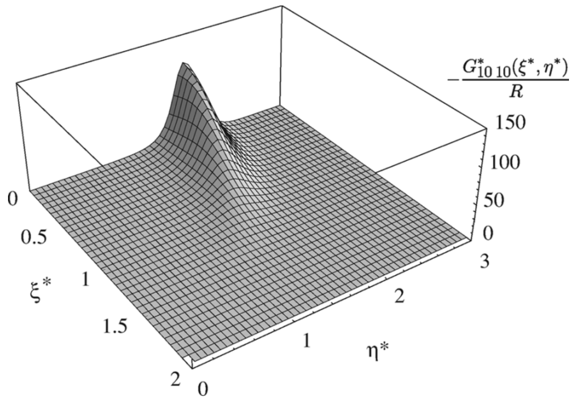

Fig. 1. Behavior of the integrand $G^*_{10\ 10}(\xi^*, \eta^*)$ for the case with $r_1 = 0.09$ and $r_2 = 0.1$.

are obtained. Decreasing the value of ε to treat the HTS as thin film, the nonlinear system does not give a convergent solution even if the adaptively deaccelerated Newton (ADNM) method [4] is adopted for the solver or integral point increase for Gauss-Legendre integration. From these reasons, we could not treat the problem with the aspect ratio condition $D/R \ll 1$ (i.e., thin film) in the previous study [5].

It is known that the double exponential (DE) formula is effective for improper integrals. By using variable transformation $\xi = -1 + 2\zeta(\xi^*)$, $\eta = 1 - 2\zeta(\eta^*)$, (10) can be rewritten in the form

$$\gamma_{ij} = \int_0^\infty d\xi^* \int_0^\infty d\eta^* G^*_{ij}(\xi^*, \eta^*) \tag{12}$$

where the function $\zeta(x)$ and $G^*_{ij}(\xi^*, \eta^*)$ are defined by

$$\zeta(x) = \exp\left(-\frac{\pi}{2}\sinh x\right) / \cosh\left(\frac{\pi}{2}\sinh x\right) \tag{13}$$

$$G^*_{ij}(\xi^*, \eta^*) = 4G_{ij}\left(-1+2\zeta(\xi^*), 1-2\zeta(\eta^*)\right) \times \frac{d\zeta(\xi^*)}{d\xi^*}\frac{d\zeta(\eta^*)}{d\eta^*}. \tag{14}$$

In Fig. 1, we show the behavior of the integrand $G^*_{10\ 10}(\xi^*, \eta^*)$ for the case with $r_1 = 0.09$ and $r_2 = 0.1$. We see from this figure, integrand $G^*_{10\ 10}(\xi^*, \eta^*)$ decreases rapidly to zero, and it can be confirmed that the singularity has disappeared. Thus, the integration region is limited to $\{(\xi^*, \eta^*) | 0 \le \xi^* \le \xi_{\rm L}, 0 \le \eta^* \le \eta_{\rm L}\}$, and γ_{ij} can be approximated as

$$\gamma_{ij} \simeq \int_0^{\xi_{\rm L}} d\xi^* \int_0^{\eta_{\rm L}} d\eta^* G^*_{ij}(\xi^*, \eta^*). \tag{15}$$

In the present study, (15) is numerically evaluated by using the trapezoid rule.

When the DE formula is adopted instead of the Gauss-Legendre formula for the numerical evaluation for the matrix W, we can obtain convergent solutions even if the condition is $D/R \ll 1$ or the Newton method with an underrelaxation is adopted for the solver.

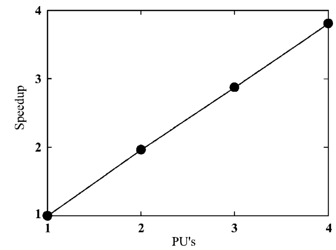

Fig. 2. Speedup ratio as function of number of processors in the case of $N = 4096$, $N_{\rm c} = 1024$, $N_{\rm g} = 32$.

TABLE I
EVALUATION ENVIRONMENT

machine	AMD Opteron 270 (Dual Core, 2GHz) x 2 = 4 processor
memory	4GB
OS	FreeBSD 6.1-STABLE
compiler	GNU fortran compiler 4.2.0 20061014
compiler option	`-O2 -fopenmp`

IV. EVALUATION

A. Parallelization

Let us first investigate the speedup ratio of the parallelization EFG for Poisson problem. The analytic 2-D region $\Omega \equiv (0,1) \times (0,1)$ is bounded by boundary. The govering equation is expressed as

$$-\Delta u = -2\pi^2 \sin \pi x \sin \pi y$$

where $u(x,y)$ denotes the unknown function and the boundary is bounded by the essential boundary condition $u = 0$.

The ratio of CPU time that each procedure spends can be investigate by profiling mode of the compiler. Note that the LU decomposition method is employed for the solver. As a result, it has been understood that CPU time used to procedure for making the coefficient matrix is about 40%. In addition, 10% is used for I/O and other procedure, and 50% is used for calculating the linear system by use of LU decomposition method.

From the above result, the speedup of procedure for making the coefficient matrix leads to the reduction of CPU time. Thus, let us parallelize the procedure for making the coefficient matrix using the OpenMP [3]. The OpenMP is a one of application programming interface (API) for multiprocessing programming in C/C++ and Fortran on a shared memory multiprocessor architecture. The OpenMP consists of a set of compiler directives, library routines, and environment variables.

Fig. 2 shows the speedup ratio of the parallelization for making the coefficient matrix as function of number of processors in the case of $N = 4096$, $N_{\rm c} = 1024$, $N_{\rm g} = 32$. Here, N, $N_{\rm c}$, and $N_{\rm g}$ denote number of node, number of cell, and integral point for Gauss-Legendre integration, respectively. Also, evaluation environment is shown in Table I. We see from this figure that, for all the cases, good performances are observed. 4PU (processing units) demonstrates 3.8 times faster performance than 1PU. From this result, it can be concluded that there is an enough parallelized effect.

Next, we parallelize the numerical code for shielding current density analysis in HTS using OpenMP. However, a good performance was not obtained. In this case, speedup ratio becomes

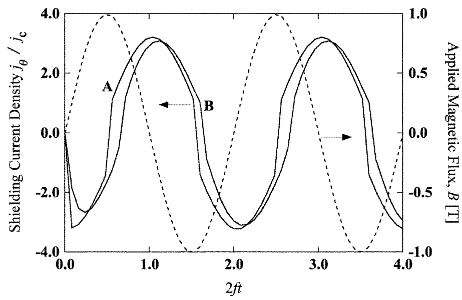

Fig. 3. Time evolution of the z component of the applied magnetic flux B_{0z} (dotted line) and the shielding current density (solid line) in case of $f = 300$ Hz, $B_0 = 1.0$ T. A: $D = 20$ μm, B: $D = 200$ nm

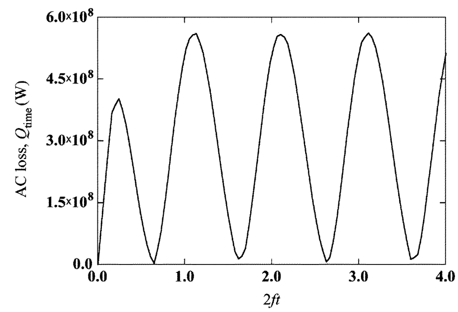

Fig. 4. Total time-dependent ac loss against nondimensional time $2ft$ in case of $f = 300$ Hz, $B_0 = 1.0$ T.

lower when number of PUs increase. This is because there are many procedures of MLS approximation in the most inner loop and various places of the code, and the MLS approximation has to evaluate linear system which dimension size is small. Therefore, it is inefficient of parallelization. Thus, we conclude that it turns to not OpenMP but Message Passing Interface. This remains as our future work.

B. Shielding Current Density and AC Loss

In this section, we apply the method given above to solve the initial boundary value problem to determine the behavior of the shielding current density in HTSs. Moreover, we calculate the ac loss from the shielding current density.

First, we show the time evolution of the shielding current density. The shielding current density of θ component is plotted as functions of nondimensional time $2ft$ in Fig. 3. The applied magnetic flux is also plotted in the same figure. Here, the value of shielding current density is calculated at $r/R = 0.9$. We can see some lagging phases between the applied magnetic flux and the shielding current density, but the value of shielding current density changes periodically and recurrently as the applied magnetic flux density changes. In other words, the shielding current density flow through the HTS to shield the applied magnetic flux. It is understood from this figure that the shielding current of $D = 200$ nm flows suddenly than that of $D = 20$ μm. This results indicate that the sudden current flowing depends on the thickness of HTS.

Next, we investigate the ac losses of the HTS. The total time-dependent ac loss is calculated by the following equation:

$$Q_{\mathrm{time}} = 4\pi\varepsilon \int_0^R r(\boldsymbol{E} \cdot \boldsymbol{j})dr. \qquad (16)$$

In Fig. 4, we show the time dependence of total ac loss. The value of ac loss changes periodically and recurrently in double-phase as the shielding current density changes.

V. Conclusion

We have developed the numerical code for analyzing the time evolution of the shielding current density in axisymmetric HTSs thin film using the EFG method. Therefore, the DE formula is adopted for the numerical evaluation for the improper integrals. The ac loss against the applied magnetic flux density is also evaluated. Furthermore, we parallelize EFG code for Poisson problem and shielding current analysis using OpenMP.

Conclusions obtained in the present study are summarized as follows.

1) To treat the shielding current analysis in HTS with the aspect ratio condition $D/R \ll 1$ (i.e., thin film) we employed double exponential formula. When the DE formula is adopted instead of the Gauss-Legendre formula for the numerical evaluation, we can obtain convergent solutions even if the condition is $D/R \ll 1$ or the Newton method with an underrelaxation is adopted for the solver.
2) We have parallelized the Poisson problem code and shielding current analysis code by using OpenMP. The results of computation shows that 4PU demonstrates 3.8 times faster than 1PU.
3) A good performance was not obtained in case of parallelized shielding current analysis code. In this case, speedup ratio becomes lower when the number of processing units increase. This is because there are many procedures of MLS approximation in the most inner loop and various places of the code, and the MLS approximation has to evaluate linear system which dimension size is small.
4) The lagging phases occur between the applied magnetic flux and the shielding current density, but the values of shielding current changes periodically and recurrently as the applied magnetic flux density changes. The value of ac loss also changes periodically and recurrently in double-phase the shielding current density changes.

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Manuscript received October 9, 2007. Corresponding author: S. Ikuno (e-mail: ikuno@nal.ikulab.org).