Parallel Hierarchical Block Wavelet Compression for an Optimal Compression Rate of 3-D BEM Problems

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Abstract—Today's low cost hardware developments allow for a parallelization of intensive computation processes over several selectable CPU cores by using Intel's OpenMP library. But if one applies this feature to a numerical simulation based on a boundary element method (BEM), there is still a huge bottle neck – the shared memory of such a system. So, if a low cost hardware should be effectively used for large BEM problems, a memory compression algorithm that is easily to be scheduled in parallel is of first choice. Within our new idea and development of the Hierarchical Block Wavelet Compression (HWC) which is based on IEEE's JPEG2000 standard for image compression, this bottle neck will be tackled in pure mathematically manners. Furthermore, its parallelization will be discussed and an optimal compression rate for a 3-D electrostatic BEM problem by a rather simple optimization algorithm will be presented.

I. Introduction

While computing a 3-D electrostatic BEM problem on a low-cost hardware several CPU cores are available. Therefore, this massively computing power is limited by a bottle neck – the shared memory.

$$u(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0} \int_A \frac{\sigma(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|} \mathrm{d}A' \tag{1}$$

Hence, a compression technique that tackles the quadratic growth of the fully populated system matrix (SM) of the system of linear equation (SLE) of (1) and also goes along with the described properties should be presented; u is the electrical potential (boundary condition) and σ is the surface charge density (unknown).

II. Parallel Hierarchical Block Wavelet Compression

The HWC is an improvement of our development of the Block Wavelet Compression (BWC) which was based on IEEE's JPEG2000 standard for digital image compression using wavelets [1]. Furthermore, the HWC allows for an arbitrary number of degree of freedom (#DoF) by setting up several squeezed and stretched Hierarchical Matrices (HMs) – wavelets are limiting the #DoFs to 2^p; as seen by BWT [2].

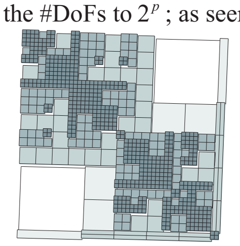

Fig. 1. HMs for an arbitrary #DoFs visualized in 3-D from C++ output

Each HM represents by its size the importance of the later computed integrals of (1) as an improvement of BWC and BWT [2] – see Fig. 1. Considered in data, the outcome is one huge list of blocks (HMs) split up automatically to the available CPU cores. To compress the BEM SM each block is computed, transformed, and thinned out in wavelet domain using a modified 2-D Wavelet Packet Transform (WPT) based on an orthonormal Haar Wavelet. Therefore, all integrals of a block have to be evaluated which results in quadratic computation costs. Whereas, the memory consumption of the whole SM is linear, due to sparse representation in compressed wavelet domains; each block. Thus, using a modified version of the 2-D WPT for block compression, allows for a matrix vector operation (MVO) at sparse SLE representation, as fast as in normal state – neither time losses nor more iteration.

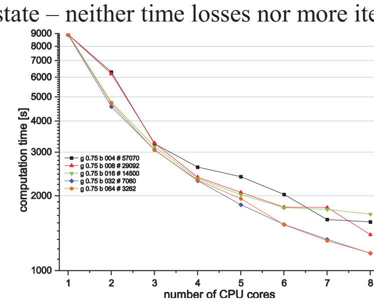

Fig. 2. E.g. for the speed up over a varying number of CPU cores

For this purpose, OpenMP performs in an effective way while having to schedule in parallel only pointer addresses to blocks for computation and compression. Also adding an object-oriented C++ implementation to the blocks using Design Patterns, the overall speedup is over 7 using 8 cores – Fig. 2.

To turn the disadvantage of quadratic computation costs to an advantage, an optimization technique should be presented that detects the optimal compression rate for each block by a requested relative error, set for the whole 3-D BEM problem.

III. Conclusion

A 3-D electrostatic BEM problem is computed in parallel by blocks representing the importance of the entries by their sizes. In the meantime each block is compressed by a self-optimizing compression algorithm using a modified 2- D WPT which allows for a rapid MVO at sparse SLE representation.

IV. References

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[2] H. F. Buchner, L. C. Wrobel, W. J. Mansur, and C. Magluta, "On the block wavelet transform applied to the boundary element method," *Eng. Analys. with Boundary Elements*, vol. 28, no. 6, pp. 571-581, 2004.

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